



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

## SOLUTION OF A PROBLEM.

BY ISAAC H. TURRELL, CUMMINSVILLE, OHIO.

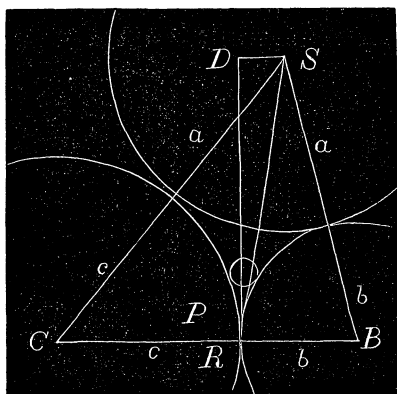
THREE circles, radii  $a, b, c$ , touch each other externally; required the radii,  $a_1, b_1, c_1$  of three circles described in the space enclosed by them, each touching the other two and two of the given circles.

A solution of this problem can be easily obtained by means of the equation proved at page 24, Vol. II. of the ANALYST.

$T^2 = 8aa_1$ ,  $T$  being the direct common tangent of the circles whose radii are  $a, a_1$  and centers  $S, S'$ ; this being a pair that do not touch each other.

Since the circles  $S, S'$ , touch  $B, C$ , externally, their center of similitude  $P$ , which is the intersection of their direct common tangents, will be in the radical axis  $DR$ . If  $N, N'$ , be the points where this common tangent, which is not shown in the figure, touches  $S, S'$ , it is well known that  $PR^2 = PN \cdot PN'$ . Again,

$$\frac{a}{a_1} = \frac{PN}{PN'} \cdot \frac{a-a_1}{a_1} = \frac{PN-PN'}{PN'} = \frac{T}{PN'}$$

$$= \frac{2\sqrt{(2aa_1)}}{PN'}, \therefore PN' = \frac{2a_1\sqrt{(2aa_1)}}{a-a_1}.$$


Similarly  $PN = \frac{2a\sqrt{(2aa_1)}}{a-a_1} \therefore PR = \frac{2aa_1\sqrt{2}}{a-a_1}.$

Draw  $SD$  perpendicular to the radical axis  $DR$ ; then  $DR$  being the altitude of the triangle  $CBS$ , its value is  $\sqrt{[abc(a+b+c)]} \div \frac{1}{2}(b+c).$

$$\text{Also } (a+b)^2 - (b-SD)^2 = DR^2 = (a+c)^2 - (c+SD)^2$$

$$\therefore SD = a(c-b) \div (c+b)$$

Again  $\sqrt{(PN^2 + a^2 - SD^2)} = PD = DR - PR$ ; whence

$$\sqrt{\left[ \frac{8a^3a_1}{(a-a_1)^2} + a^2 - \frac{a^2(c-b)^2}{(c+b)^2} \right]} = \frac{2\sqrt{[abc(a+b+c)]}}{b+c} - \frac{2aa_1\sqrt{2}}{a-a_1},$$

an equation of the second degree in  $a_1$ .

Squaring this equation, reducing, and solving with reference to  $a_1$ ,

$$a_1 = \frac{a \{ bc+ac+ab+\sqrt{[2abc(a+b+c)]} \} \pm a^2(b+c) \pm a \sqrt{[2abc(a+b+c)]}}{bc+2ab+2ac+2\sqrt{[2abc(a+b+c)]}}$$

whence  $a_1 = a$  or  $\frac{1}{1+m-1 \div a}$ , where  $\frac{1}{m} = \frac{2}{a} + \frac{2}{b} + \frac{2}{c} + 2\left(\frac{2}{ab} + \frac{2}{bc} + \frac{2}{ca}\right)^{\frac{1}{2}}$

Similarly  $b_1 = \frac{1}{1+m-1 \div b}$ ,  $c_1 = \frac{1}{1+m-1 \div c}$ , the radii required.

The first value of  $a_1$  is worthy of note; for if it were required to determine the position of four circles  $b, c, b_1, c_1$  of given magnitude, that touch each other consecutively, so that a circle could be drawn touching all, the equation  $a_1 = a$ , shows that each of these circles fulfills this condition.

Again,  $a, b, c$ , being the radii of the first, we have the following relations connecting the radii of the circles of the various groups.

$$\left. \begin{array}{l} \frac{1}{a_1} = \frac{1}{m} - \frac{1}{a} \\ \frac{1}{b_1} = \frac{1}{m} - \frac{1}{b} \\ \frac{1}{c_1} = \frac{1}{m} - \frac{1}{c} \end{array} \right\} \text{2nd,} \quad \left. \begin{array}{l} \frac{1}{a_2} = \frac{1}{m_1} - \frac{1}{a_1} \\ \frac{1}{b_2} = \frac{1}{m_1} - \frac{1}{b_1} \\ \frac{1}{c_2} = \frac{1}{m_1} - \frac{1}{c_1} \end{array} \right\} \text{3rd,} \quad \left. \begin{array}{l} \frac{1}{a_x} = \frac{1}{m_{x-1}} - \frac{1}{a_{x-1}} \\ \frac{1}{b_x} = \frac{1}{m_{x-1}} - \frac{1}{b_{x-1}} \\ \frac{1}{c_x} = \frac{1}{m_{x-1}} - \frac{1}{c_{x-1}} \end{array} \right\} x+1.$$

Now substituting for  $1 \div a_1, 1 \div b_1, 1 \div c_1$  in the third group, their values as given in the second, and carrying the resulting values of  $1 \div a_2, 1 \div b_2, 1 \div c_2$  into the fourth, and so on, to the  $(x+1)$ th group, we find that

$$\frac{1}{a_x} + \frac{1}{a} = \frac{1}{b_x} + \frac{1}{b} = \frac{1}{c_x} + \frac{1}{c} = \frac{1}{m_{x-1}} - \frac{1}{m_{x-2}} + \dots + \frac{1}{m},$$

when  $x$  is odd, and

$$\frac{1}{a_x} - \frac{1}{a} = \frac{1}{b_x} - \frac{1}{b} = \frac{1}{c_x} - \frac{1}{c} = \frac{1}{m_{x-1}} - \frac{1}{m_{x-2}} + \dots - \frac{1}{m}$$

when  $x$  is an even number.

## A PROBLEM IN SURVEYING.

BY T. J. LOWRY, M. S., SAN FRANCISCO, CALIFORNIA.

*Problem*.—Required the positions of the two places of observation  $y$  and  $m$ , with reference to three known points  $A, B$  and  $C$ , having observed at  $m$  the angles  $AmB$  and  $Bmy$  and at  $y$  the angles  $ByA$  and  $Cym$ .

*Trig. Analysis*.—In the isosceles  $\triangle ABe$  we have the base  $AB$  and  $\angle AeB (=2AmB)$ , and hence all the  $\angle$ s to find  $Ae$  or  $Be$ . And in  $\triangle Ade$  are known  $\angle Aed (=180^\circ - AmB)$ , side  $Ae$ , and  $\angle Ade (=AyB)$  to get  $Ad$  and  $de$ . Now in isosceles  $\triangle Age$  having sides  $Ae$  and  $ge$ , and  $\angle Aeg [=$

